EV charging with a smooth profile

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Learning objectives

At the end of this chapter you will be able to:

- penalize the changes of an aggregate quantity with a quadratic term;
- read the energy-requirement multiplier in a QP;
- trace the frontier between spend and jitter of the draw.

Prerequisites

- From the module *LP Modelling*, Chapter 5 «Smart charging of electric vehicles»: the linear charging model, with its data
- From this module, Chapter 2 «Background: convexity, quadratic programming and KKT conditions»: convex quadratic programs, the KKT conditions and the multipliers as shadow prices

What changes

The case and its limit

- Case, data and linear model: LP Modelling, "Smart charging of electric vehicles"
- Minimum-cost charging chases the cheap hours: peak of 103.4 kW, jumps of up to 54.9 kW from one hour to the next
- Steep ramps strain the grid and are often paid for
- Problem class: **convex QP**

The idea

- A **quadratic** penalty on the hourly jumps of the total draw $z_t = b_t + \sum_v x_{vt}$
- z_t is a linear expression, not a new variable
- A single new datum: $\gamma \in \mathbb{Q}_{\geq 0}$ (€ per kW^2)
- The base load enters the penalty: charging can soften its ramps

The model

QP model of charging with a smooth profile

QP model of charging with a smooth profile

$$\min \quad \sum_{v=1}^{n}\sum_{t=1}^{m} \pi_t \, \Delta t \, x_{vt} + \gamma \sum_{t=2}^{m} \left(z_t - z_{t-1}\right)^2 \tag{10.1a}$$

$$\text{subject to} \quad \eta \sum_{t=1}^{m} \Delta t \, x_{vt} \geq e_v, \qquad \forall v \in \{1, \ldots, n\}, \tag{10.1b}$$

$$\sum_{v=1}^{n} x_{vt} + b_t \leq k, \qquad \forall t \in \{1, \ldots, m\}, \tag{10.1c}$$

$$x_{vt} \leq a_{vt} \, \bar{p}_v, \qquad \forall v, \forall t, \tag{10.1d}$$

$$x_{vt} \geq 0, \qquad \forall v, \forall t. \tag{10.1e}$$

Worked example

One vehicle, two hours

Instance

$e = 10$ kWh, $\eta = 1$, $\pi_1 = 0.10$, $\pi_2 = 0.20$ €/kWh, $\bar{p} = 12$ kW, zero base load

Linear model: $x_1 = 10$, spend € 1.00

- Step 1, with $\gamma = 0.01$: $\tilde{x} = (6.25;\ 3.75)$, $\tilde{z} = 1.4375$, $\tilde{\lambda} = 0.15$
- Step 2: $x_1 = 5 + d$, $x_2 = 5 - d$, $f(d) = 1.5 - 0.1d + 4\gamma d^2 \Rightarrow$ $d = 1/(80\gamma)$

The multiplier is the average of the prices

- Step 3: stationarity

$$0.10 - 2\gamma(x_2 - x_1) = \lambda = 0.20 + 2\gamma(x_2 - x_1) \;\Rightarrow\; \lambda = 0.15$$

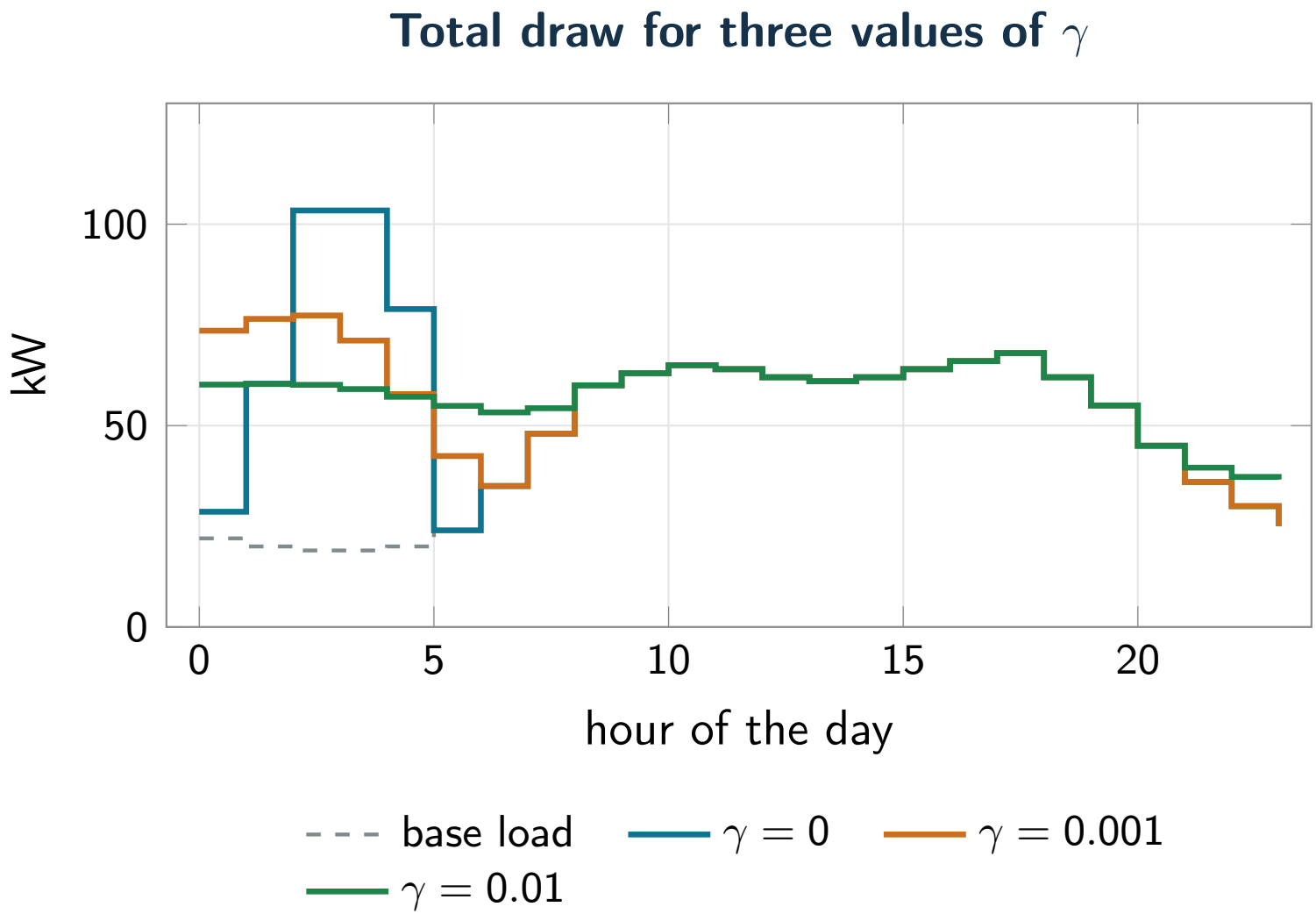
- It holds for every $\gamma \geq 1/400$ (as long as $x_2 \geq 0$)
- In the linear model the marginal kWh cost the price of the marginal hour (0.10)

Implementation and results

Only the objective changes

```
z = {t: x.sum("*", t) + base[t] for t in ore}
salti = gp.quicksum((z[t] - z[t-1])**2
                    for t in ore[1:])
m.setObjective(spesa + gamma*salti, GRB.MINIMIZE)
```

Three values of γ



Spend, peak and jumps

```
gamma = 0.000: spend 20.36  peak 103.4 kW  largest jump 54.9
gamma = 0.001: spend 21.76  peak  77.3 kW  largest jump 15.4
gamma = 0.010: spend 24.67  peak  68.0 kW  largest jump 10.0
```

- With $\gamma = 0.01$: € 4.31 more per night, peak at the lowest possible value (68 kW)
- The linear compromise of LP Modelling only looks at the peak; the QP softens every ramp

Sensitivity analysis

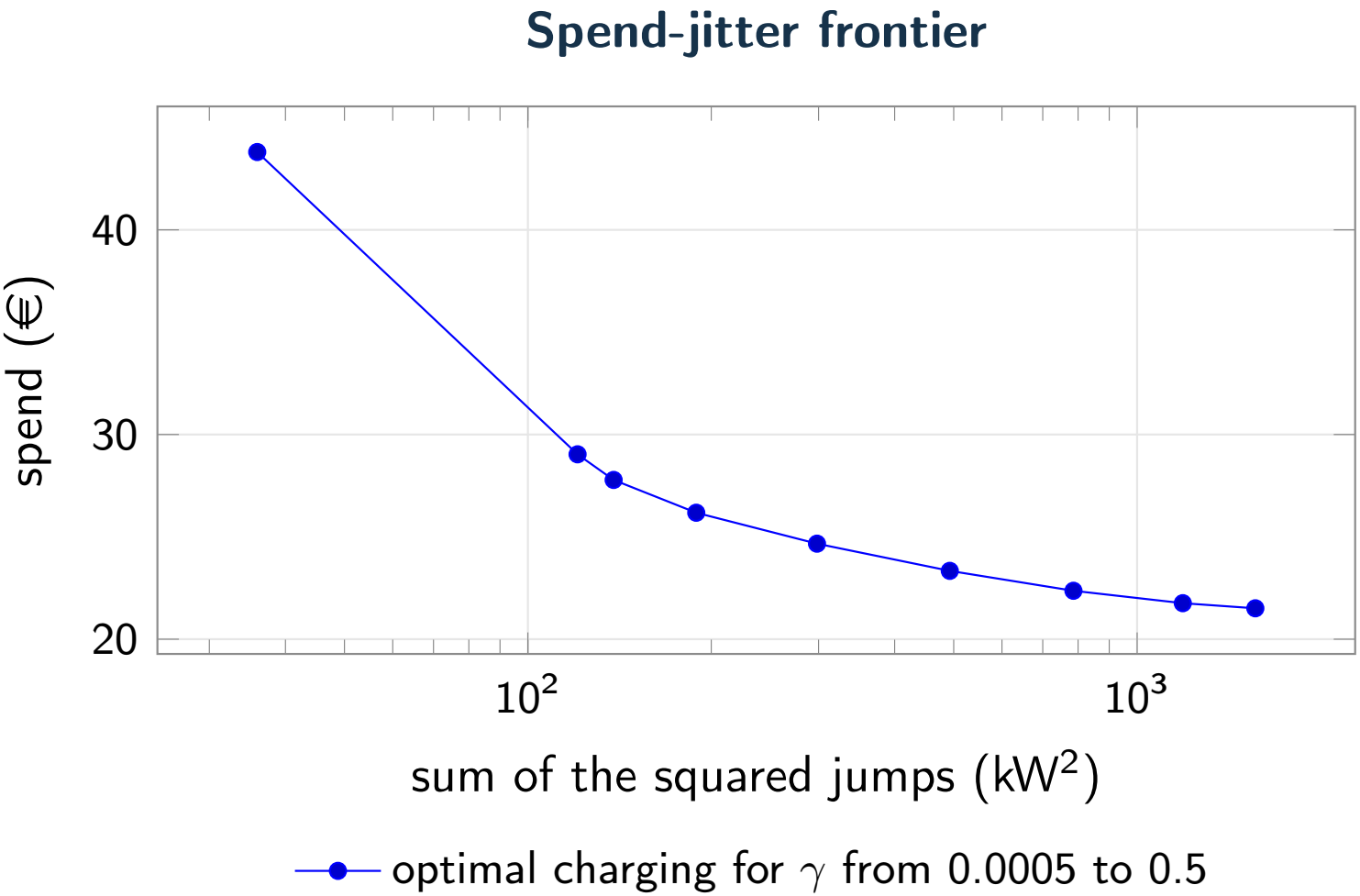
A single price for all vehicles

```
            V1       V2       V3       V4       V5       V6
g=0      0.0947   0.0842   0.0842   0.0947   0.0947   0.0842
g=0.01   0.0916   0.0916   0.0916   0.0916   0.0916   0.0916
Check (g=0.01): +1 kWh -> 0.0917; +0.01 -> 0.0916
```

- The penalty links the vehicles through the total draw: the multipliers coincide

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The spend-jitter frontier



- With € 1.15 more the sum of the squared jumps falls from 7,268 to 1,563

Exercises

Exercises

1. **By hand**: example with $\gamma = 0.05$
2. **Check**: multiplier of V3 with $\gamma = 0.001$
3. **Charging only**: penalize the jumps of $\sum_v x_{vt}$
4. **Penalty on power**: $\gamma \sum_t z_t^2$, $\gamma = 0.001$
5. **Meter**: $k = 70$ kW with $\gamma = 0.001$

Summary

- The quadratic term penalizes every change, not only the peak
- The marginal kWh is split between hours: in the example it costs the average price
- In the QP the multipliers of the vehicles coincide
- The frontier is steep at first: the first smoothing steps cost little