Smart charging of electric vehicles

Outline

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Learning objectives (1/2)

At the end of this chapter you will be able to:

- model decisions over an hourly horizon with availability windows;
- read the duals as prices of the marginal hours;

Learning objectives (2/2)

At the end of this chapter you will be able to:

- handle the cost-peak trade-off with a weighted objective;
- diagnose a real pitfall of sensitivity analysis: the right-hand side with constants.

Prerequisites

- From this module, Chapter 2 «Background: linear programming and sensitivity analysis»: shadow prices and the sensitivity protocol
- From this module, Chapter 3 «The solver: building, solving, interpreting»: variable bounds in `gurobipy` and the right-hand side stored by the solver

Managerial motivation

Introduction: Managerial motivation

- **Problem class:** LP
- We look at a new operational problem, created by the energy transition
- We will understand why a decision that *seems* to be of on/off type is in fact modelled with continuous variables

The fleet problem

- A company fleet must be fully charged by the morning by exploiting the cheap night hours
- But if all the vehicles charge together during the cheap hours, the **peak draw** explodes
- With it explode the power costs and the risk of exceeding the capacity of the meter
- The model decides *how much power* to give to each vehicle in each hour: a naturally **continuous** decision

Smart charging

- Smart charging is one of the new operational problems created by the energy transition
- Ingredients: electric fleets, hourly energy prices, power limits of the meters
- A perfect example of a problem that *seems* to require on/off variables and instead is a **pure LP**
- The reason: the real decision — how much power — is continuous

Summary

- Goal: charge the fleet at night at minimum cost
- Risk: concentrating all the charging in the cheap hours creates a peak of draw
- The decision is **how much power** per vehicle and per hour: continuous variables, no binary variables

Guided construction of the model

Introduction: Guided construction of the model

- We describe the problem in words, then fix data and variables
- We write the minimum-cost charging LP model and explain every family of constraints
- Finally we see the variants: peak shaving and multi-objective

The problem in words

- *We decide* how much power to deliver to each vehicle in each hour
- *The objective* is to minimize the energy expenditure of the night
- *The constraints*:
  - every vehicle must receive all the energy it needs before departure
  - it can charge only while it is plugged in and never above the power of its charger
  - the overall draw (charging plus building) can never exceed the power of the meter

Data (input of the model) (1/2)

Symbol	Type	Meaning
n	$\in \mathbb{Z}_{\geq 1}$	number of vehicles, indexed by $v \in \{1, 2, \ldots, n\}$
m	$\in \mathbb{Z}_{\geq 1}$	number of hourly intervals, indexed by $t \in \{1, 2, \ldots, m\}$, of duration Δt (here $\Delta t = 1$ h)
π_t	$\in \mathbb{Q}_{\geq 0}$	energy price in hour t (€/kWh)
a_{vt}	$\in \{0, 1\}$	observed *datum*: 1 if vehicle v is plugged in during hour t, 0 otherwise
e_v	$\in \mathbb{Q}_{>0}$	energy required by vehicle v before departure (kWh)

Data (input of the model) (2/2)

Symbol	Type	Meaning
$\bar{p}_v$	$\in \mathbb{Q}_{>0}$	maximum charging power of vehicle v (kW)
b_t	$\in \mathbb{Q}_{\geq 0}$	base load of the building in hour t (kW)
k	$\in \mathbb{Q}_{>0}$	maximum power that can be drawn from the meter (kW)
η	$\in \mathbb{Q},\ 0 < \eta \leq 1$	charging efficiency

Decision variables

We introduce the following $n\,m$ non-negative variables:

$$x_{vt} = \text{power assigned to vehicle } v \text{ in hour } t \text{ (kW)},$$

$$\forall v \in \{1, 2, \ldots, n\},\ \forall t \in \{1, 2, \ldots, m\}.$$

Using these variables, we now write an LP model for the problem.

Minimum-cost charging LP model

Minimum-cost charging LP model

$$\min \quad \sum_{v=1}^{n} \sum_{t=1}^{m} \pi_t \, \Delta t \, x_{vt} \tag{5.1a}$$

$$\text{subject to} \quad \eta \sum_{t=1}^{m} \Delta t \, x_{vt} \geq e_v, \qquad \forall v \in \{1, 2, \ldots, n\}, \tag{5.1b}$$

$$\sum_{v=1}^{n} x_{vt} + b_t \leq k, \qquad \forall t \in \{1, 2, \ldots, m\}, \tag{5.1c}$$

$$x_{vt} \leq a_{vt} \, \bar{p}_v, \quad \forall v \in \{1, 2, \ldots, n\}, \ \forall t \in \{1, 2, \ldots, m\}, \tag{5.1d}$$

$$x_{vt} \geq 0, \qquad \forall v \in \{1, 2, \ldots, n\}, \ \forall t \in \{1, 2, \ldots, m\}. \tag{5.1e}$$

Objective function and energy constraints

- The linear objective function minimizes the energy expenditure of the night: the power x_{vt} delivered for the duration Δt costs $\pi_t \, \Delta t \, x_{vt}$ euros
- The energy constraints ensure that every vehicle receives all the energy it requires before departure (n linear constraints)
- The efficiency η stands on the left: of the energy that is drawn, only the fraction η ends up in the battery

Grid constraints, power constraints and domain

- The grid constraints impose that the overall draw (charging plus base load) never exceeds the power of the meter: they **couple the vehicles** to each other (m linear constraints)
- The power constraints limit every vehicle to its charger and set the power to zero in the hours in which the vehicle is absent ($a_{vt} = 0$) ($n\,m$ linear constraints)
- Availability is a *datum*, not a decision: this is why **no binary variables are needed**
- The constraints $x_{vt} \geq 0$ define the variables of the model

Variants of the model

- **Peak shaving**: $\min z$ with

$$\sum_{v=1}^{n} x_{vt} + b_t \leq z, \qquad \forall t \in \{1, 2, \ldots, m\}$$

- **Multi-objective**: cost $+ \rho\, z$

The energy constraint written as $\geq$

- Charging more than necessary is allowed
- But it is never worthwhile: the price is positive
- Hence at the optimum the energy constraint holds **with equality**

Summary

- Continuous variables x_{vt}: power per vehicle and per hour
- Objective: energy expenditure $\sum_v \sum_t \pi_t \, \Delta t \, x_{vt}$
- Constraints: energy per vehicle, meter capacity per hour, charger power and availability
- Availability a_{vt} is a datum: the model remains a pure LP

Worked example by hand

Introduction: Worked example by hand

- We solve by hand a minimal case: one vehicle, two hours
- We compute the optimal solution and the shadow prices
- We obtain the key for reading the whole chapter: the dual is the price of the marginal hour

Example: one vehicle, two hours (1/3)

- We need $e = 10$ kWh ($\eta = 1$)
- Prices: $\pi_1 = 0.10$ and $\pi_2 = 0.20$ €/kWh
- Maximum power: $\bar{p} = 8$ kW

Example: one vehicle, two hours (2/3)

Case $\bar{p} = 8$.

- In the cheap hour we charge at the maximum (8 kWh); the remaining 2 kWh go into the expensive hour
- Cost: $8 \cdot 0.10 + 2 \cdot 0.20 = 1.20$ €
- *Shadow price of the energy requirement*: the eleventh kWh would end up in the expensive hour $\Rightarrow$ 0.20 €/kWh
- *Shadow price of the power limit* in hour 1: one more kW would allow us to move one kWh from the expensive hour to the cheap one $\Rightarrow$ 0.10 €/kW

Example: one vehicle, two hours (3/3)

Case $\bar{p} = 12$.

- Everything in the cheap hour: cost 1.00 €
- Dual of the requirement: 0.10
- Dual of the power: 0 (constraint not active)

The pattern "**the dual is the price of the marginal hour**" is the key for reading the whole chapter.

Summary

- The cheap hours are filled first, up to the power limit
- The shadow price of the energy requirement is the price of the marginal hour
- The shadow price of the power limit is the saving obtained by moving energy from the expensive hour to the cheap one; it is zero if the constraint is not active

Case study

Introduction: Case study

- We apply the model to a realistic fleet
- We see how the model is written in Python with Gurobi
- We get ready to read the results and the shadow prices

The data of the case study

- Six vans with different night charging windows (data in `dati/ev_flotta.csv`)
- Stylized hourly prices with an evening peak
- Base load of the building
- A meter of $k = 120$ kW, efficiency $\eta = 0.95$

The model in Gurobi (1/2)

Variables, with the power limit and the availability as upper bound:

```
x = m.addVars(veicoli, ore, name="x")
for v in veicoli:
    for t in ore:
        # 0 if not plugged in
        x[v, t].UB = P[v] * disp[v, t]
```

The model in Gurobi (2/2)

Energy and grid constraints, objective function:

```
v_ene  = m.addConstrs((eta * gp.quicksum(x[v, t]
                          for t in ore)
                          >= E[v] for v in veicoli),
                         name="energia")
v_rete = m.addConstrs((gp.quicksum(x[v, t]
                          for v in veicoli)
                          + base[t] <= C for t in ore),
                         name="rete")
m.setObjective(gp.quicksum(prezzo[t] * x[v, t]
                for v in veicoli for t in ore),
                GRB.MINIMIZE)
```

Summary

- Six vans, different windows, a meter of 120 kW, $\eta = 0.95$
- The power constraint simply becomes the upper bound of the variable (0 if the vehicle is not plugged in)
- Energy and grid are two families of constraints with `addConstrs`

Results

Introduction: Results

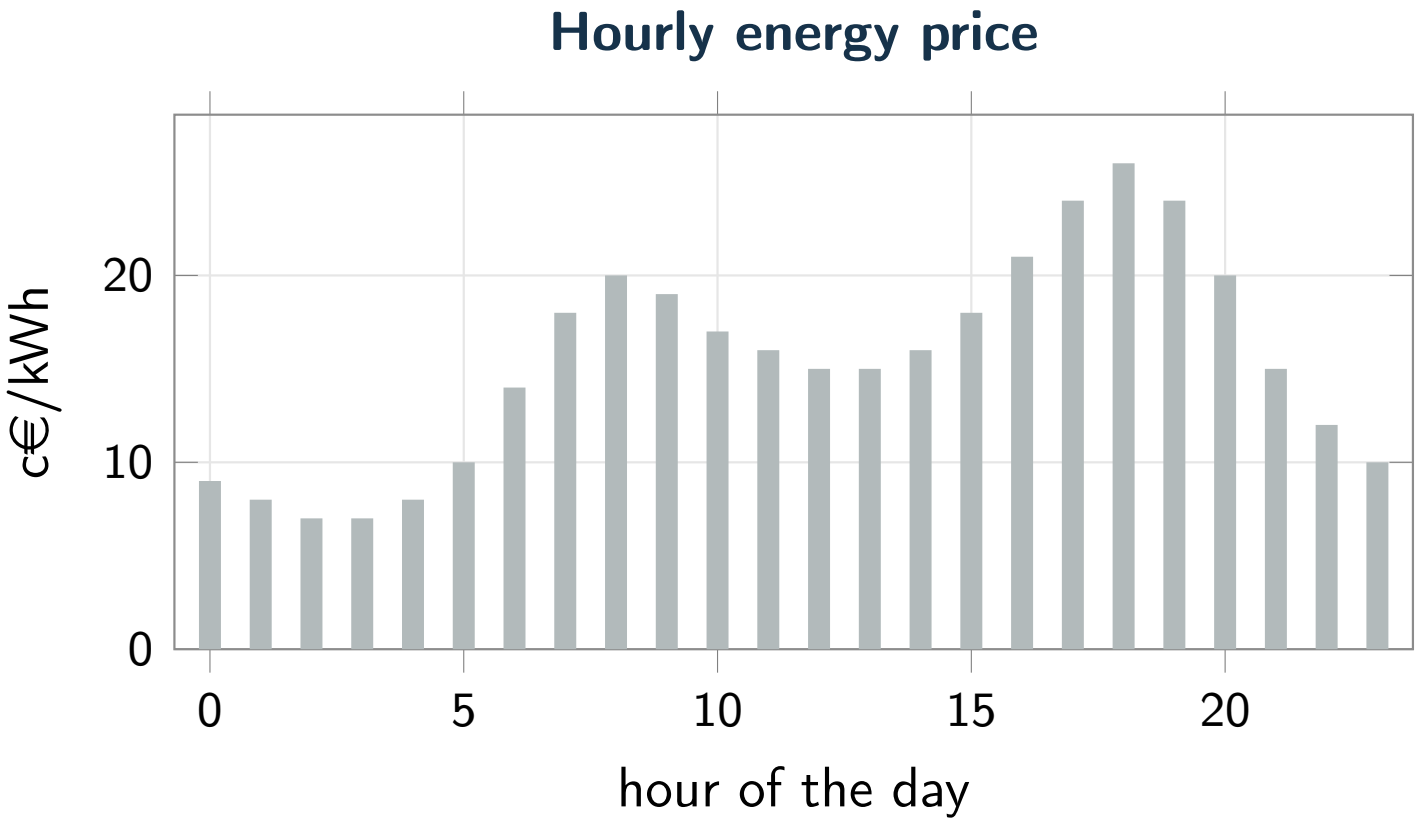
- We compare the minimum-cost strategy with peak shaving
- We look at the draw profiles hour by hour
- We interpret the shadow prices of the energy requirement as prices of the marginal hours

Output of the model

```
Minimum cost : 20.36 EUR/night
               peak draw 103.4 kW (limit 120)
Peak shaving : minimum possible peak 68.0 kW

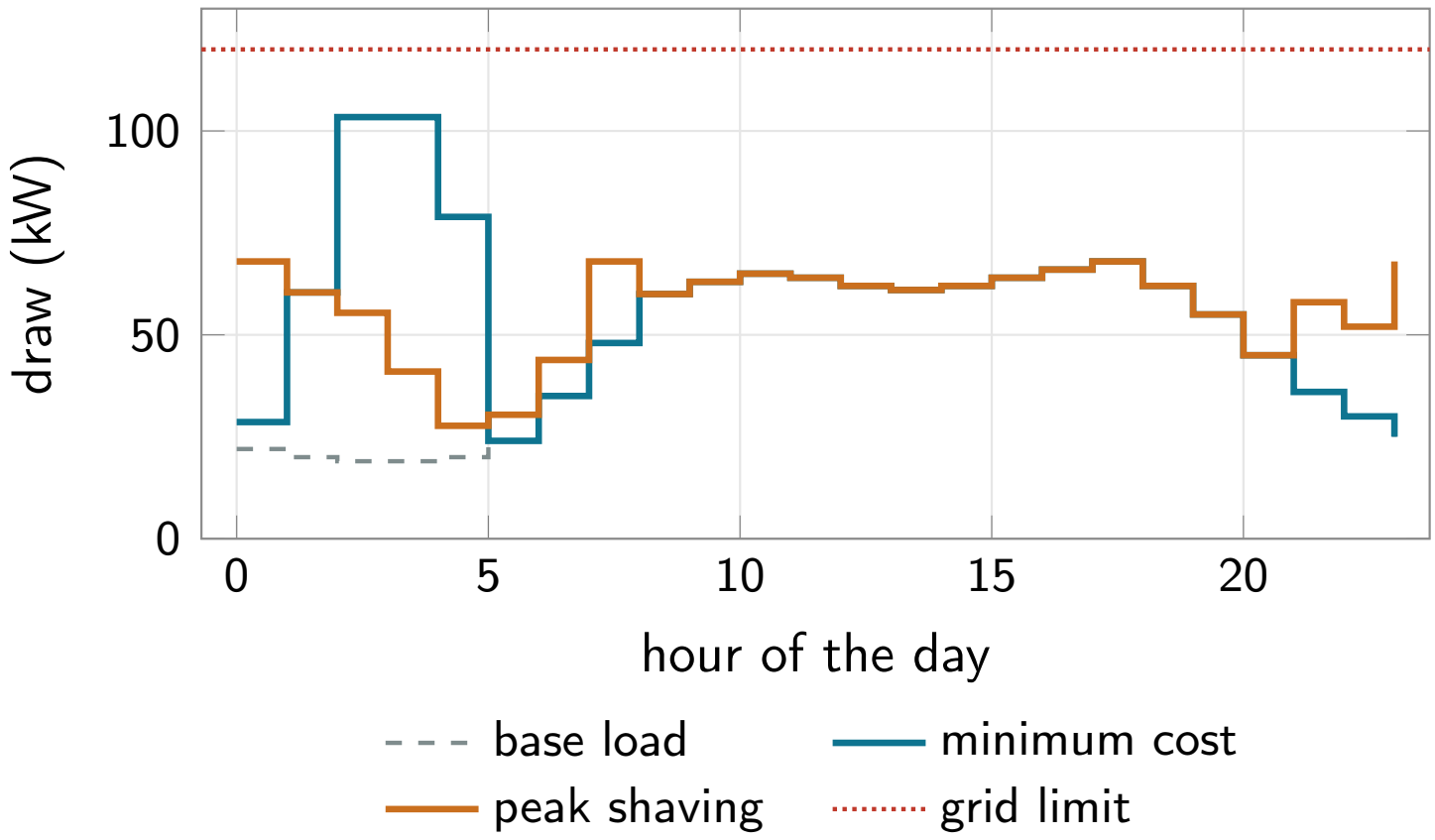
Shadow prices of the energy requirement (EUR/kWh):
  V1 0.0947   V2 0.0842   V3 0.0842
  V4 0.0947   V5 0.0947   V6 0.0842
```

Hourly energy price



The price is low at night and has an evening peak.

Total draw: chasing the prices creates a night-time peak



Reading the chart

- The chart shows the total draw (base + charging) hour by hour under the two strategies
- The minimum cost one concentrates the charging in the cheap hours and creates a night-time peak of 103 kW
- Peak shaving spreads the charging out and keeps the draw at 68 kW
- Both stay below the grid limit of 120 kW

The duals tell the story of the marginal hours (1/2)

- The shadow prices of the energy requirement are equal to

$$0.0842 = 0.08/\eta \qquad \text{or} \qquad 0.0947 = 0.09/\eta$$

- They are the prices of the *marginal hours* of each vehicle (the cheapest hour still free within its window), corrected for the efficiency

The duals tell the story of the marginal hours (2/2)

- V2, V3 and V6 have windows that cover the hours at 0.08
- V1, V4 and V5 arrive late or leave early: their marginal hour costs 0.09
- One additional kWh of requirement therefore costs between 8.4 and 9.5 cents depending on the vehicle

Summary

- Minimum cost: 20.36 €/night with a peak of 103.4 kW
- Peak shaving: minimum possible peak 68 kW
- Shadow prices of the requirement = price of the marginal hour/η: 8.4 or 9.5 cents per kWh

Sensitivity analysis

Introduction: Sensitivity analysis

- We study the trade-off between cost and peak by varying the weight ρ of the peak
- We vary the capacity k of the meter
- We see an instructive bug, which really happened, in the modification of the right-hand side

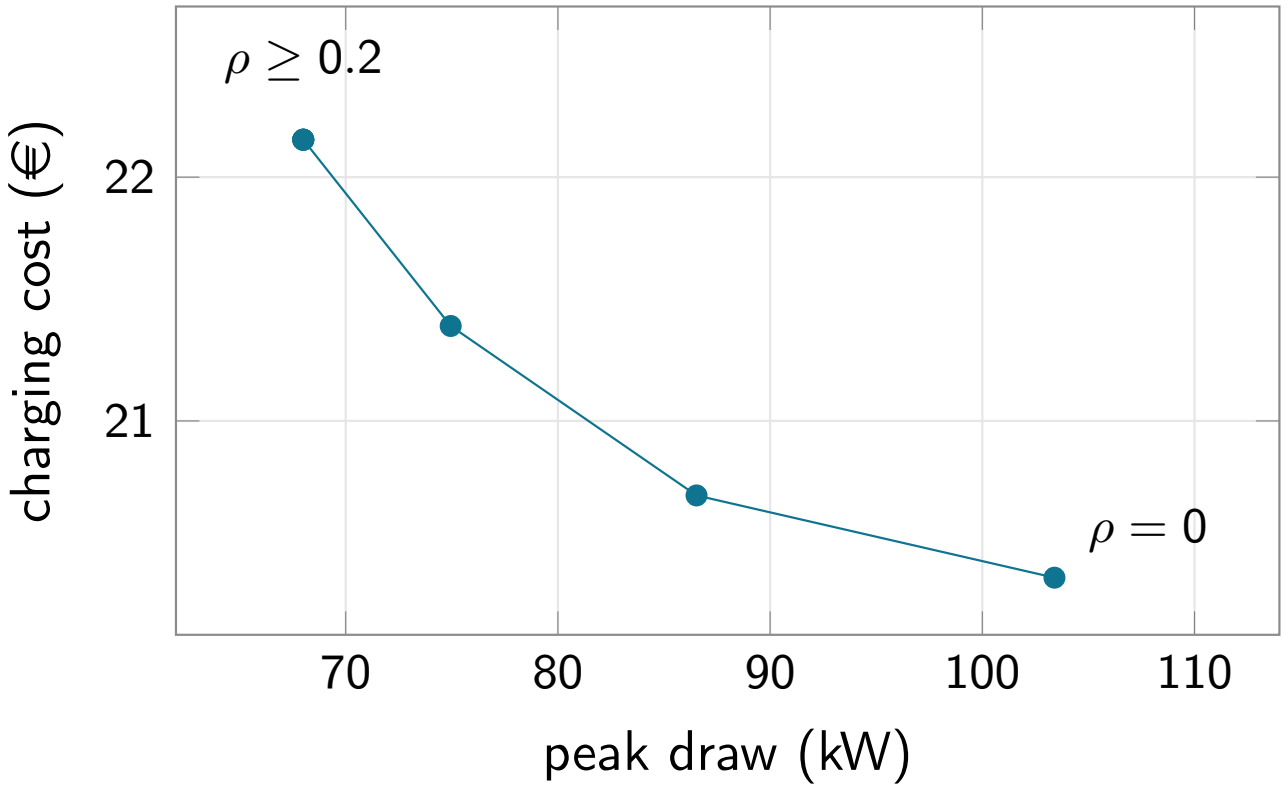
Trade-off cost + ρ·peak

```
Trade-off cost + rho*peak:
  rho = 0.00: cost 20.36  peak 103.4
  rho = 0.05: cost 20.69  peak  86.5
  rho = 0.10: cost 21.39  peak  74.9
  rho = 0.20: cost 22.15  peak  68.0
  rho = 0.50: cost 22.15  peak  68.0
```

Meter capacity

```
Meter capacity:
  k = 60, 65 kW: INFEASIBLE
  k = 70 kW    : 21.93
  k = 80 kW    : 21.09
  k = 90 kW    : 20.63
  k = 120 kW   : 20.36
```

Cost-peak frontier



Reading the frontier

- The frontier is obtained by varying the weight ρ of the peak in the objective
- Every point is an optimal charging strategy
- Reducing the peak from 103 to 75 kW costs about one euro per night
- The point $\rho \geq 0.2$ reaches the minimum possible peak (68 kW) at the lowest cost that allows it

Peak shaving pays for itself

- Cutting the peak from 103 to 75 kW (-28%) costs only one euro per night; getting down to 68 kW costs less than two
- If the power tariff or the upgrade of the meter cost more than that, peak shaving pays for itself
- The pure minimax (peak 68) without the cost term would spend 27.79 €
- The trade-off with $\rho = 0.2$ achieves *the same peak* at 22.15 €: **never optimize a single objective when there are two of them**

An instructive bug, which really happened (1/2)

Observation 1: An instructive bug, which really happened

In the constraint `quicksum(x) + base[t] <= C`*, Gurobi moves the constant* `base[t]` *into the right-hand side: the stored RHS is* `C - base[t]`*.*

The first version of our sensitivity analysis wrote `v.RHS = nuovaC` *and obtained identical costs for every capacity.*

An instructive bug, which really happened (2/2)

Observation 2: An instructive bug, which really happened

It was in fact relaxing the constraint by `base[t]`.

The correct form is `v.RHS = nuovaC - base[t]`.

When a sensitivity analysis changes nothing, suspect your own code before suspecting the model.

Summary

- The weight ρ traces the cost-peak frontier: with about one euro per night the peak drops from 103 to 75 kW
- Below 70 kW of meter capacity the problem becomes infeasible
- The constants in the constraints end up in the RHS: modify it taking them into account

The quadratic variant

- A **smooth** load profile penalizes the changes of the load from one hour to the next
- It needs a quadratic term in the objective: the model is **no longer an LP**
- It is the quadratic variant of this model, in the module *NLP Modelling*: "Electric vehicle charging with a smooth profile"

Exercises

Introduction: Exercises

- Four exercises to put the model to the test
- Checking the duals, changes in the windows, emissions, minimum capacity
- They are solved by modifying the Python script of the chapter

Exercise: check

Verify by perturbation the shadow price of the energy requirement of V1:

- raise e_1 from 46 to 47 kWh
- explain why it is equal to 0.09/0.95

Exercise: late arrival

V3 arrives at 23 instead of 20:

- how do cost and peak change?
- which shadow price increases?

Exercise: emissions

- With an hourly carbon intensity g_t (high in the evening, low at night), draw the cost-emissions frontier
- Are the cheap hours also the cleanest ones in this case?

Exercise: minimum capacity

- Find by bisection the minimum capacity k that keeps the problem feasible
- Explain which combination of windows and $\bar{p}_v$ determines it

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Summary (1/2)

Model:

- Smart charging is a pure LP: the decision is how much power, availability is a datum
- Energy constraints, grid constraints (they couple the vehicles) and power constraints

Duals:

- The shadow price of the requirement is the price of the marginal hour, divided by η

Summary (2/2)

Multi-objective:

- Cost $+ \rho$ peak traces the frontier; the trade-off reaches the minimum peak at a cost much lower than the pure minimax

Sensitivity:

- The constants of the constraints end up in the RHS: if a sensitivity analysis changes nothing, check the code first